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MAGNETO-FLUID DYNAMICS DIVISION

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THE BOUNDARY LAYER IN THE CONFINEMENT OF A ONE-DIMENSIONAL PLASMA

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and H. Weitzner

July 15, 1962

AEC Research and Development Report

NEW YORK UNIVERSITY

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Abstract

An attempt is described to confine a plasma composed of electrons and ions by static, one-component, electric and magnetic fields. All quantities depend on one space coordinate only, and the particle acceleration vector has two components. The contained plasma is assumed to be at rest with no electric or magnetic fields present. For the major portion of the report, the electron and ion temperatures are assumed equal and no trapped particles are permitted. For both species of particles the Vlasov equation is used.

A charge-neutral approximation is discussed together with the exact solution, and numerical results are presented. It is shown on one hand that under the given assumptions the boundary layer cannot be charge-neutral throughout, and on the other hand that even the exact solution does not correspond to a desirable confinement configuration, as it contains a runaway ion jet and large electric fields.

Finally, two simple, albeit impractical, cases are presented, one with unequal ion and electron temperatures, and the other with trapped particles, in which a containment situation is indeed obtained.

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The Boundary Layer in the Confinement of a One-Dimensional Plasma

I. Introduction

In this report we describe attempts to obtain a simple configuration in which a plasma composed of both ions and electrons is contained by static electric and magnetic fields.[1] We consider the simplest non-trivial situation in which the one-component fields depend on one space coordinate only and the particle acceleration vector has only two non-vanishing components. We assume the contained plasma to be at rest and in thermal equilibrium with equal ion and electron temperatures. We also assume that no fields are present in the contained plasma. We require that there are no trapped particles in the boundary layer between the confined plasma and the vacuum; that is, we demand that all particle orbits start and end in the confined plasma. We assume that the particles interact with each other only through the average field produced by all the particles; that is, we employ the Vlasov equation for both species.

The next section contains the explicit formulation of the problem and a detailed description of the trapped particles. We discuss the "charge-neutral" approximation and then give certain expansions useful in the numerical treatment of this approximation and the exact solution. The results of a numerical computation of the charge-neutral curve are in

1. For earlier work directly related to this problem but without electric field, see H. Grad, Phys. Fluids 4, 1366 (1961).

the third section. We conclude that a boundary layer without trapped particles cannot be essentially charge neutral throughout the layer. In the fourth section we present the numerical solution to the full problem. We find that the only solution is physically unacceptable as it contains a run-away ion jet and large electric fields. We are thus forced to consider more complicated containment configurations and in the last section we present two simple schemes, one with different electron and ion temperatures, and the other with trapped particles. We offer these examples only to show that some kind of confinement is indeed possible; we do not propose them as physically interesting -- or attainable -- states.

II. Formulation of the Problem

Consistent with the conditions of the previous section, we assume that the electric and magnetic fields vary with the x coordinate only, that the electric field has an x component only, that the magnetic field has a z component, and that the non-trivial velocity components are u in the x direction and v in the y direction. The Vlasov equation for the distribution function of each species of particles (+ ions, - electrons) becomes

$$(1) \qquad u \frac{\partial f_\pm}{\partial x} (x,u,v) + \frac{e_\pm}{m_\pm} \left\{ (E + vB) \frac{\partial f_\pm}{\partial u} - uB \frac{\partial f_\pm}{\partial v} \right\} = 0 \ ,$$

and the fields satisfy Maxwell's equations which, in rationalized MKS units are

$$(2a) \qquad \kappa \frac{\partial E(x)}{\partial x} = e(n_+(x) - n_-(x))$$

$$(2b) \qquad -\frac{1}{\mu}\frac{\partial B(x)}{\partial x} = e(J_+(x) - J_-(x)) \ ,$$

where

$$(3a) \qquad n_{\pm}(x) = \int f_{\pm}(x,u,v)dudv \ ,$$

and

$$(3b) \qquad J_{\pm}(x) = \int vf_{\pm}(x,u,v)dudv \ .$$

In order to solve the Vlasov equation and to introduce the concept of trapped particles, we define the characteristic coordinates of (1) by

$$(4a) \qquad \frac{dx(s)}{ds} = u(s)$$

$$(4b) \qquad \frac{du(s)}{ds} = \frac{e}{m}\left(E(x(s)) + v(s)B(x(s))\right)$$

$$(4c) \qquad \frac{dv(s)}{ds} = -\frac{e}{m}u(s)B(x(s)) \ ,$$

so that

$$(5) \qquad \frac{d}{ds} f(x(s), u(s), v(s)) = 0 \ .$$

Equation (5) states the well known result that $f(x,u,v)$ is constant on the trajectories of the particles in the given fields. There are two sets of equations (4) and (5), one set for the ions and one for the electrons; however, we temporarily suppress the distinction. We know that the solution of the equations (4) has the exact constants of the motion

$$\varepsilon = \frac{1}{2} m(u^2 + v^2) + e\Phi \tag{6a}$$

$$P = mv + eA \ , \tag{6b}$$

where we have introduced the usual scalar and vector potentials by

$$\frac{d\Phi(x)}{dx} = -E(x) \tag{7a}$$

$$\frac{dA}{dx} = +B(x) \ . \tag{7b}$$

In a certain sense we might say that a class of solutions to equation (5) is any function $f(\varepsilon,P)$.

In the problem we are considering we assume the distribution function to be Maxwellian as $x \rightarrow -\infty$, or

$$\underset{x \longrightarrow -\infty}{f(x,u,v)} = \frac{n_o m}{2\pi kT} \exp\left(-\frac{1}{2} m \frac{(u^2 + v^2)}{kT}\right) \ .$$

As we suppose that $\Phi(x)$ and $A(x)$ tend to zero as x tends to minus infinity, we see that

$$(8) \qquad \underset{x \to -\infty}{f(x,u,v)} = \frac{n_o m}{2\pi kT} \exp\left(-\frac{\varepsilon}{kT}\right) .$$

One might infer from the above remarks that (8) holds everywhere in the plasma, but this need not be the case. Certain trajectories, defined by (4), need not reach $x = -\infty$; thus the "boundary condition" (8) will not determine $f(x,u,v)$ on those orbits. We call such particles trapped particles, and throughout the main portion of this work we assume that no trapped particles are present. From (6) we see that

$$m^2 u^2 = 2m(\varepsilon - e\Phi) - (P - eA)^2 ,$$

so that for u to be real,

$$(9) \qquad 2m(\varepsilon - e\Phi) - (P - eA)^2 \geq 0 .$$

Given a particle at a point x with energy ε and canonical momentum in the y direction P, the above inequality must hold at all points to the left of its position (i.e., for all $x' \leq x$) if the particle is to be able to reach (or to come from) the point at $-\infty$, or, in other words, for the particle not to be trapped. For such untrapped particles we assume (8) holds; otherwise we take $f(x,u,v) = 0$.

We may now easily eliminate equation (1) entirely by expressing $n_\pm(x)$ and $J_\pm(x)$ in terms of $A(x)$ and $\Phi(x)$. After a change of variables from u and v to ε and P we obtain

from the basic definitions (3), the equations

$$\text{(10a)} \qquad n(x) = \frac{n_o}{\pi kT} \iint\limits_{D(x)} \frac{d\varepsilon dP \; e^{-\varepsilon/kT}}{\sqrt{2m(\varepsilon - e\Phi(x)) - (P-eA(x))^2}}$$

$$\text{(10b)} \qquad J(x) = \frac{n_o}{m\pi kT} \iint\limits_{D(x)} \frac{d\varepsilon dP \; e^{-\varepsilon/kT} \, (P-eA(x))}{\sqrt{2m(\varepsilon - e\Phi(x)) - (P-eA(x))^2}}$$

where the domain of integration $D(x)$ is defined by the restriction that it contains all values of ε, $0 < \varepsilon < \infty$ and all values of P, $-\infty < P < \infty$, such that

$$\text{(11)} \qquad 2m(\varepsilon - e\Phi(x') - (P-eA(x'))^2 \geq 0$$

for all $x' \leq x$. In order to proceed further we must recognize the difference between the ions and electrons. For later reference we non-dimensionalize (10) and (11) for ions and electrons separately.

We introduce non-dimensional potentials by

$$\text{(12a)} \qquad \phi_\pm(x) = \frac{e_\pm \; \Phi(x)}{kT_\pm}$$

$$\text{(12b)} \qquad a_\pm(x) = \frac{e_\pm A(x)}{\sqrt{2m_\pm kT_\pm}} \quad .$$

On performing the ε integration first, and with the definitions

$$(13) \qquad F(x) = e^{x^2} \int_x^\infty e^{-y^2} dy$$

and

$$(14) \qquad \varepsilon_\pm(p,x) = \sup_{x' \leq x} \left\{ \phi_\pm(x') + (p - a_\pm(x'))^2 \right\} ,$$

we obtain

$$(15a) \qquad n_\pm(x) = \frac{2n_o}{\pi} \int_{-\infty}^{\infty} dp \; F\left(\sqrt{\varepsilon_\pm(p,x) - \phi_\pm(x) - (p - a_\pm(x))^2}\right) e^{-\varepsilon_\pm(p,x)}$$

$$(15b) \qquad J_\pm(x) = \frac{2n_o}{\pi} \sqrt{\frac{2kT_\pm}{m_\pm}} \int_{-\infty}^{\infty} dp\,(p - a_\pm(x)) e^{-\varepsilon_\pm(p,x)} \cdot F\left(\sqrt{\varepsilon_\pm(p,x) - \phi_\pm(x) - (p - a_\pm(x))^2}\right) ,$$

the non-dimensional variable of integration p being defined as $P/\sqrt{2km_\pm T_\pm}$ respectively for the ion and electron case. The system of equations is now (2) and (15).

In order to complete the non-dimensionalization of the full system, we define

$$(16a) \qquad \gamma = \left(\frac{m_-}{m_+}\right)^{1/4}$$

$$(16b) \qquad T = T_+ \;\; (= T_- \text{ by assumption})$$

$$(16c) \qquad a(x) = -\gamma a_-(x) = \frac{1}{\gamma} a_+(x)$$

(16d) $$\phi(x) = \phi_+(x) = -\phi_-(x)$$

(16e) $$x = \frac{1}{e}\sqrt{\frac{\sqrt{m_-m_+}}{\mu n_o}}\;\xi$$

and the equations become

(17a) $$\rho_\pm(\xi) = \frac{2}{\pi}\int_{-\infty}^{\infty} dp\; G_\pm(p,\xi)e^{-\varepsilon_\pm(p,\xi)}$$

(17b) $$j_\pm(\xi) = \frac{2}{\pi}\int_{-\infty}^{\infty} dp\; G_\pm(p,\xi)e^{-\varepsilon_\pm(p,\xi)}(p \mp \gamma^{\pm 1}\, a(\xi))$$

(18a) $$\frac{d^2}{d\xi^2}\, a(\xi) = \frac{1}{\gamma}\, j_-(\xi) - \gamma j_+(\xi)$$

(18b) $$\frac{\kappa\mu kT}{(m_-m_+)^{1/2}}\,\frac{d^2}{d\xi^2}\,\phi(\xi) = \rho_-(\xi) - \rho_+(\xi),$$

where

(19a) $$\varepsilon_\pm(p,\xi) = \sup_{\xi'\leq\xi}\left\{\pm\phi(\xi') + (p \mp \gamma^{\pm 1}\, a(\xi'))^2\right\}$$

and

(19b) $$G_\pm(p,\xi) = F\left(\sqrt{\varepsilon_\pm(p,\xi) \mp \phi(\xi) - (p \mp \gamma^{\pm 1}\, a(\xi)^2}\right) .$$

The dimensionless constant in (18b) is just $\frac{1}{2}\,(\overline{v}/c)^2$, where $\overline{v}$ is the geometrical mean of the ion and electron thermal speeds and c is the speed of light.

Before solving the full system of differential equations we consider an approximation of both mathematical and physical meaning. Since $\overline{v}/c$ should be much less than one, it is reasonable to assume that the solution of (18b) is given approximately by the solution to

$$(20) \qquad 0 = \rho_+(\xi) - \rho_-(\xi) \; .$$

Equation (20) states that the plasma is charge-neutral, and physically we expect that the plasma should try to remain in an approximately charge-neutral state. Mathematically we expect that the solution to (18) could be developed in an asymptotic expansion in powers of $\kappa\mu kT/\sqrt{m_-m_+}$ and that the leading term would be given by the solution to (20). Since there is no explicit ξ dependence in (20), we may reinterpret the equation as a definition of an implicit functional relation between ϕ and a. With this functional relation known we might solve the one remaining differential equation (18a). We shall obtain by numerical means the functional relation between ϕ and a for various values of γ. We do not proceed to the solution of the remaining differential equation as we would not obtain any further information of interest.

In order to check and to start the numerical work we have obtained an expansion of the solution to (20), the charge-neutral curve, for a and ϕ small. After relatively straightforward manipulations we derive:

(21) $$\phi = -\ell_o a + m a^{3/2} + n a^{7/4} + O(a^2) ,$$

where ℓ_o is the unique non-negative zero of

(22a) $$f(\ell) = \sqrt{2\ell} \int_0^\infty \sqrt{x}\, dx\, e^{-(x - \frac{\ell}{2\gamma})^2} - \sqrt{\frac{2}{\gamma}} \int_{-\infty}^0 \sqrt{-x}\, dx\, e^{-(x - \frac{\ell\gamma}{2})^2}$$

and

(22b) $$m = - \frac{\ell_o \pi}{f'(\ell_o)} < 0$$

and

(22c) $$n = - \frac{3}{2} \frac{m\sqrt{-m}}{f'(\ell_o)} e^{-(\ell_o/2\gamma)^2} \left(\frac{6}{5} (\sqrt{6} - 2) - \frac{4}{9}\right) .$$

In formulas (21), (22), we assume $\gamma \leq 1$ (as it is physically). One may also verify as easily that there is no other root of (20) of the form $\phi = \ell a^n + \dots$, $n > 0$, starting from $\phi = a = 0$. In the next section we give the full numerical solution to (20).

In order to begin the solution to the full differential equations (18a) and (18b), we need starting values of ϕ, $\frac{\partial\phi}{\partial\xi}$, a, $\frac{\partial a}{\partial\xi}$; moreover, we must show that there is a solution leaving the singular point $\phi = a = 0$ of the equations (18). If we look for a solution which, for small ϕ and a, has the expansion

(23) $$\phi = -\ell a + \dots , \qquad (\ell \geq 0) ,$$

then we obtain from (18a) and (18b) the expansions

$$\text{(24a)} \qquad \frac{d^2 a(\xi)}{d\xi^2} = \alpha(\ell)\sqrt{a(\xi)} + \dots ,$$

$$\text{(24b)} \qquad -\ell\left(\frac{\kappa\mu kT}{\sqrt{m_- m_+}}\right)\frac{d^2 a(\xi)}{d\xi^2} = \frac{2}{\pi}\sqrt{a}\, f(\ell) + \dots ,$$

where

$$\text{(25)} \qquad \alpha(\ell) = \frac{1}{\pi}\left\{(2\gamma)^{3/2}\int_0^\infty \sqrt{x}\,\left(x - \frac{\ell}{2\gamma}\right)dx\, e^{-\left(x-\frac{\ell}{2\gamma}\right)^2} - \left(\frac{2}{\gamma}\right)^{3/2}\int_{-\infty}^0 dx\,\sqrt{-x}\,\left(x-\frac{\ell\gamma}{2}\right)e^{-\left(x-\frac{\ell\gamma}{2}\right)^2}\right\},$$

and $f(\ell)$ is defined by (22a). From (24a) we infer that

$$\text{(26)} \qquad a(\xi) = \left(\frac{\alpha(\ell)}{12}\right)^2(\xi-\xi_o)^4 + \dots , \qquad \xi > \xi_o$$

$$= 0 , \qquad \xi < \xi_o .$$

As we may choose ξ_o arbitrarily, we set $\xi_o = 0$. If we insert this expansion in (24b) we obtain the relation

$$\text{(27)} \qquad f(\ell) = -\frac{\pi}{2}\,\ell\alpha(\ell)\,\frac{\kappa\mu kT}{\sqrt{m_- m_+}} .$$

For $\kappa\mu kT/\sqrt{m_- m_+}$ sufficiently small we infer the existence of a root of (27) near the value ℓ_o at which $f(\ell_o) = 0$. Thus we

have shown the existence of a solution to the full differential equations which leaves the singular point $\phi = a = 0$. The form of this solution is given by the relations (23), (26), and (27). We do not assert that this solution is unique; however, there is no other solution of the type (23), and with some effort one can prove that there is no solution of the form $\phi = \pm \ell a^n + \dots$, $n > 0$. Thus we have chosen to ignore some solutions in which electric effects are larger than magnetic ones and badly behaved solutions, e.g., those with oscillations of period tending to zero as a and ϕ tend to zero. We note further that the solution obtained is in fact near the solution given by the charge-neutral curve. In the fourth section we describe the numerical solution of (18).

III. The Charge-Neutral Curve

Before describing the results of the computation of the solution of (20), we explain briefly the techniques used. The function $F(x)$ appearing in the definition of the charge and current densities satisfies the differential equation

$$F'(x) = -1 + 2xF(x) \tag{28a}$$

and has the integral representation

$$F(x) = \frac{1}{2}\int_0^\infty \frac{e^{-u}du}{\sqrt{u+x^2}} \quad . \tag{28b}$$

We computed $F(x)$ for one large value of x from (28b) and then integrated (28a) inward to set up a table of values of $F(x)$. Since, for $x \geq 0$, we are integrating in a stable direction we expected no errors to propagate and we found that the error in the table of values was no larger than a few times 10^{-8}. Besides tabulating $F(x)$, we also stored the first three derivatives of $F(x)$, so that we could use Taylor's formula for interpolation. Thus the total error in the computation of $F(x)$ was no larger than a few times 10^{-8}. The only real complication in the computation of the solution of (20) comes from the definition of $\varepsilon_{\pm}(p,x)$ by (19a). We stored tables of values of the variable of integration p in (17a) and the associated values of $\varepsilon_{\pm}(p,x)$. After solving for $a(x)$ for a given $\phi(x)$, we corrected the values of $\varepsilon_{\pm}(p,x)$ according to (19a), and then we increased $\phi(x)$ and searched for a solution of $\rho_+ - \rho_- = 0$. In the searching procedure the first guess was a linear extrapolation of the previous results or the value given by (21). If this value was not sufficiently close to the root, then we tried another and finally interpolated among the various values until a sufficiently precise root was found. We discovered that unless $\rho_+ - \rho_-$ was quite small (not much larger than 10^{-7}) cumulative errors could destroy the solution. Further, we found that in critical sections of the curve a relatively small increment in $\phi(x)$ each step was also needed. Except for these difficulties the numerical computation was quite direct. We might add,

however, that the computing time for the charge-neutral curve was almost as long as the time to obtain the solution to the full differential equations.

We computed at least part of the charge-neutral curve for three mass ratios, $m_-/m_+ = \frac{1}{1836}, \frac{1}{100}, \frac{1}{10}$. The computations were easiest and most reliable for the smallest mass ratio. In Figure 1 we plot ϕ and $n_+ (= n_-)$ as a function of a for the mass ratio $\frac{1}{1836}$. The curves for the other mass ratios were qualitatively the same as in Figure 1, but while the curves turned back, we could not be sure of the details of the turn-around since the finite net of points necessary for the integrals (17a) limited the ultimate accuracy in this very sensitive region of the curve. In another connection in the next section we give the corresponding curve for a mass ratio of $\frac{1}{100}$.

We considered earlier the possibility of expanding the solution of the full differential equations in an asymptotic series in the parameter $\kappa\mu kT/\sqrt{m_- m_+}$, and the first term of this series was to come from the charge-neutral solution. We infer from Figure 1 the impossibility of such an expansion throughout the full boundary layer, for

$$E = -\frac{\partial\phi}{\partial x} = -\frac{\partial\phi}{\partial a}\frac{\partial a}{\partial x} = -\frac{\partial\phi}{\partial a} B , \tag{29}$$

and we see that $\partial\phi/\partial a$ is infinite at the turn-around point. Thus we are forced to turn to a numerical solution of the full

differential equations in order to discover what happens to the solution in this region.

IV. The Solution to the Full Differential Equations

In order to solve the system of differential equations (18), we used the methods of the previous section to compute $\rho(x)$ and $j(x)$ and then the modified Runge-Kutta method to advance the solution one step. Then we adjusted the quantities $\varepsilon_{\pm}(p,x)$ to include the new values and proceeded with the integration. We checked the results primarily by comparing with the charge-neutral solution and also by recomputing then with different initial conditions and different meshes. With the meshes chosen none of the quantities changed more than a few percent in any interval (except at the very start of the solution where its behavior is given by (26)). We shall examine several characteristics of the solutions obtained.

We checked first that the solution we obtained did in fact follow the charge-neutral curve initially. In Figure 2 we plot ϕ as a function of a for $m_-/m_+ = \frac{1}{100}$ for several values of the ratio $\overline{v}/c$; we also include the charge neutral curve for comparison. In Figure 2 we see exactly the behavior we would expect if the charge neutral curve is to be the leading term of an asymptotic expansion of the full solution: as the

expansion parameter becomes smaller and smaller the solution approaches the charge-neutral curve.

In figures 3, 4, and 5 we present the results of the numerical computations for various values of the parameters involved. In Figure 3, $m_-/m_+ = \frac{1}{100}$, $\bar{v}/c\sqrt{2}) = \frac{1}{50}$; in Figure 4, $m_-/m_+ = \frac{1}{100}$, $\bar{v}/(c\sqrt{2}) = \frac{1}{10}$; and in Figure 5, we chose $m_-/m_+ = \frac{1}{1836}$ and $\bar{v}/c\sqrt{2}) = \frac{1}{50}$. We have chosen the peculiar variable

$$(30) \qquad \tilde{E}(x) = E(x) \frac{\kappa\mu kT}{2\sqrt{m_- m_+}}$$

for then the electromagnetic part of the pressure tensor assumes the simple form

$$(31) \qquad P_{xx}^{\text{electromagn.}} = \frac{1}{2}\left(B^2 - \hat{E}^2\right) .$$

Thus we see that after the solution leaves the charge-neutral curve the electrons turn back very rapidly and only the ions remain. The resulting net positive charge causes the electric field to increase as x increases, and this in turn accelerates the ions forward. Since the magnetic field still turns back particles the ion density drops. However, the ion pressure must continue to rise to balance the electric field pressure. In other words, we find evidence of a type of runaway phenomenon, with a jet of highly energetic ions leaving the plasma. This solution obviously is not acceptable physically

as it requires electric fields whose pressure is much larger than the magnetic pressure. Thus the conclusion that we offer is that there is no steady-state confinement with equal ion andelectron temperature without trapped particles in the boundary layer. In the next section we show that by relaxing either of two of the conditions imposed we can find a confinement situation.

V. Other Solutions.

If we relax the condition that both species of particles must be at the same temperature, then we may easily obtain a solution with $\phi = E = 0$. If we consider a situation in which

$$(31) \qquad T_-/T_+ = m_+/m_- \; ,$$

then we see from (12) and (15) that, since $a_+(x) = -a_-(x)$, the assumption

$$(32) \qquad \phi = E = 0$$

is consistent with the necessary condition, $n_+(x) = n_-(x)$. Hence the equations reduce to the magnetic ones alone and the solution becomes the one found by H. Grad[1], after a simple

1. See footnote, page 4.

rescaling of parameters appropriate to a system with mass

$$\frac{1}{m} = \frac{1}{m_+} + \frac{1}{m_-} \tag{33a}$$

and temperature

$$T = T_+ + T_- \ . \tag{33b}$$

We may find another type of confinement for a plasma with equal ion and electron temperatures by the addition of trapped particles. Again we look for a solution with $\phi = E = 0$. Hence we must demand that $\rho_+ = \rho_-$ everywhere. We note that since there are no electric fields, zero velocity particles are stationary. Hence we can add an arbitrary density of trapped particles of zero velocity at any point in the plasma. In particular, we may consider the introduction of zero-velocity electrons, in order to match the net positive charge of the free particles. Since the trapped electrons introduced are at rest, they do not contribute to the current; hence equation (18a) still holds. The condition of charge neutrality obviously determines the density of the trapped electrons, and

$$\rho_-^{\text{trapped}}(x) = \rho_+^{\text{free}}(x) - \rho_-^{\text{free}}(x) \ . \tag{34}$$

Details of such a solution are shown in Figure 6 for $\frac{m_-}{m_+} = \frac{1}{100}$. We see that the electrons turn back rapidly and that the ions

turn back much more slowly. The characteristic size of such a layer is thus the ion larmor radius. We do not claim physical relevance for these solutions; we offer them only to show that the problem of confinement is really a question of picking the correct solution out of many possible ones.

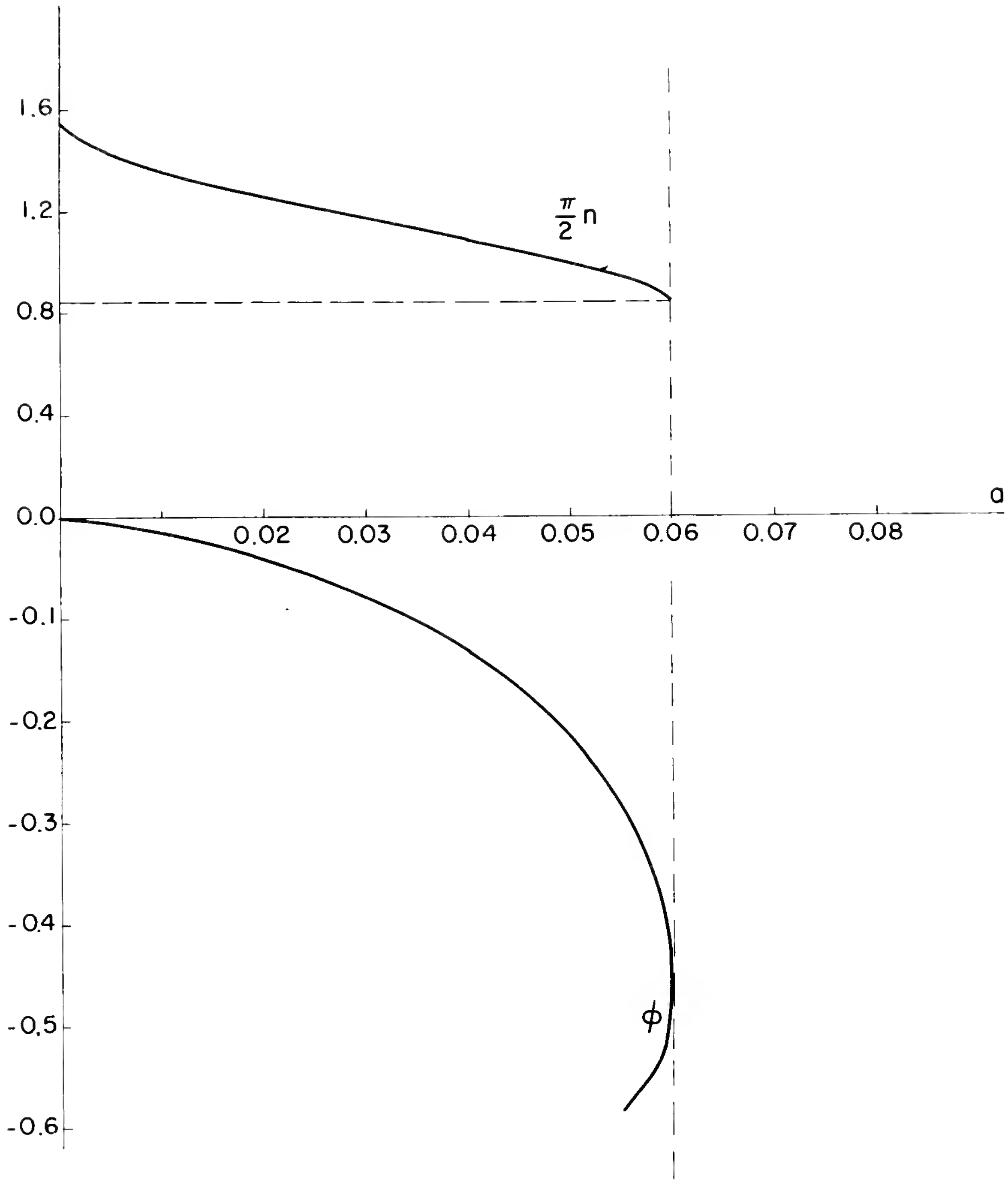

Figure I. Charge neutrality curve for mass ratio = 1/1836.

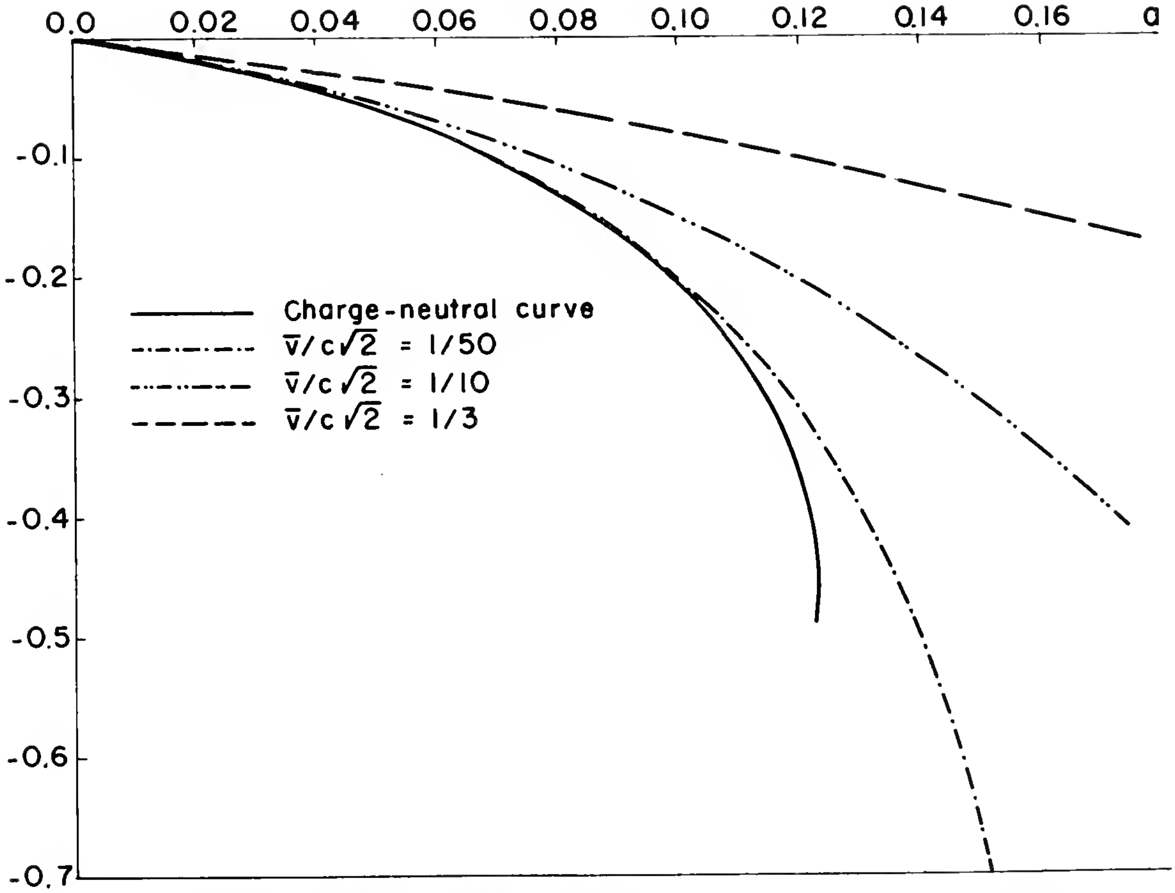

Figure 2. Dependance of the solution on $\bar{v}/c$ for the case $m_-/m_+ = 1/100$

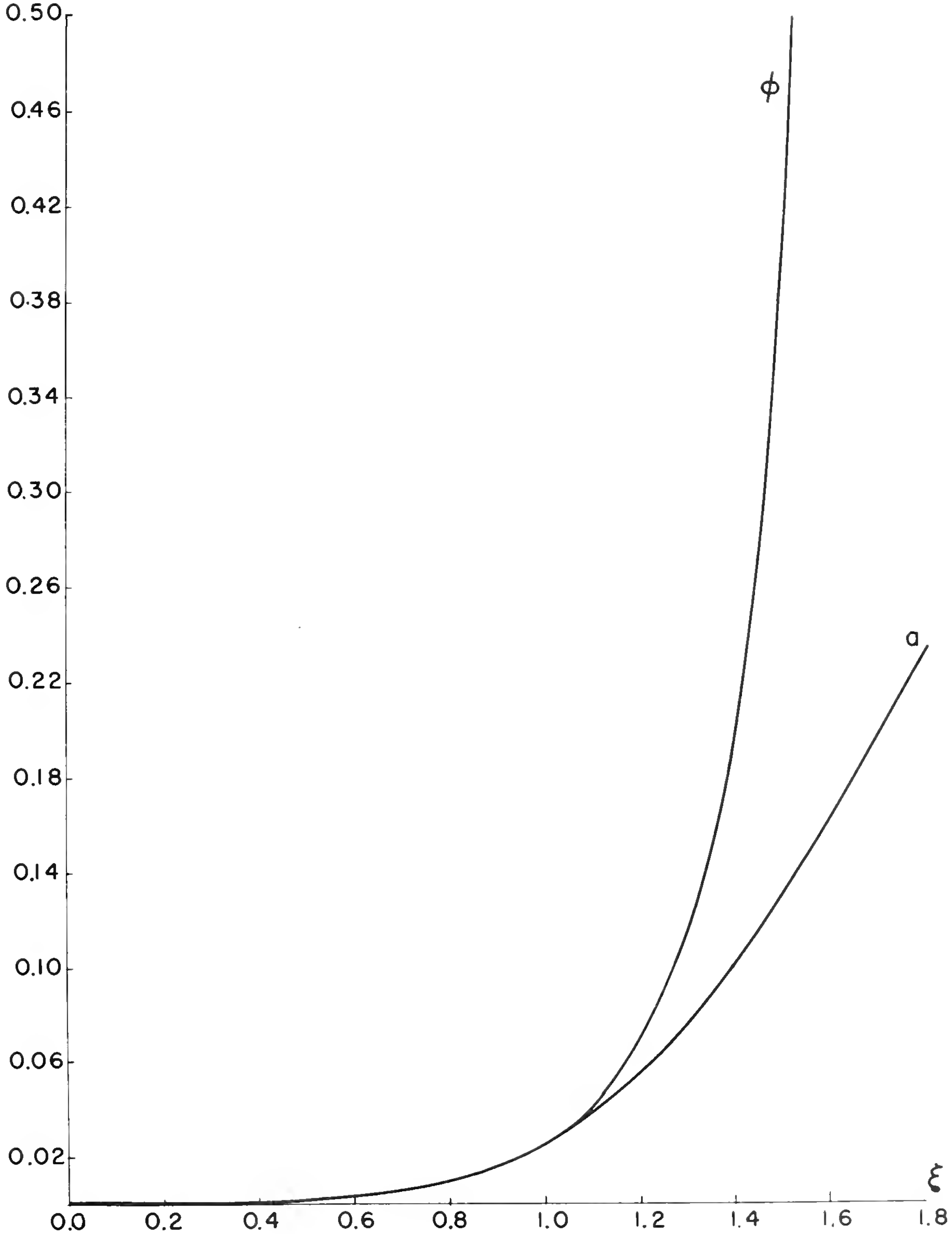

Figure 3a. Solution for $m_-/m_+ = 1/100$; $\bar{v}/c\sqrt{2} = 1/50$.

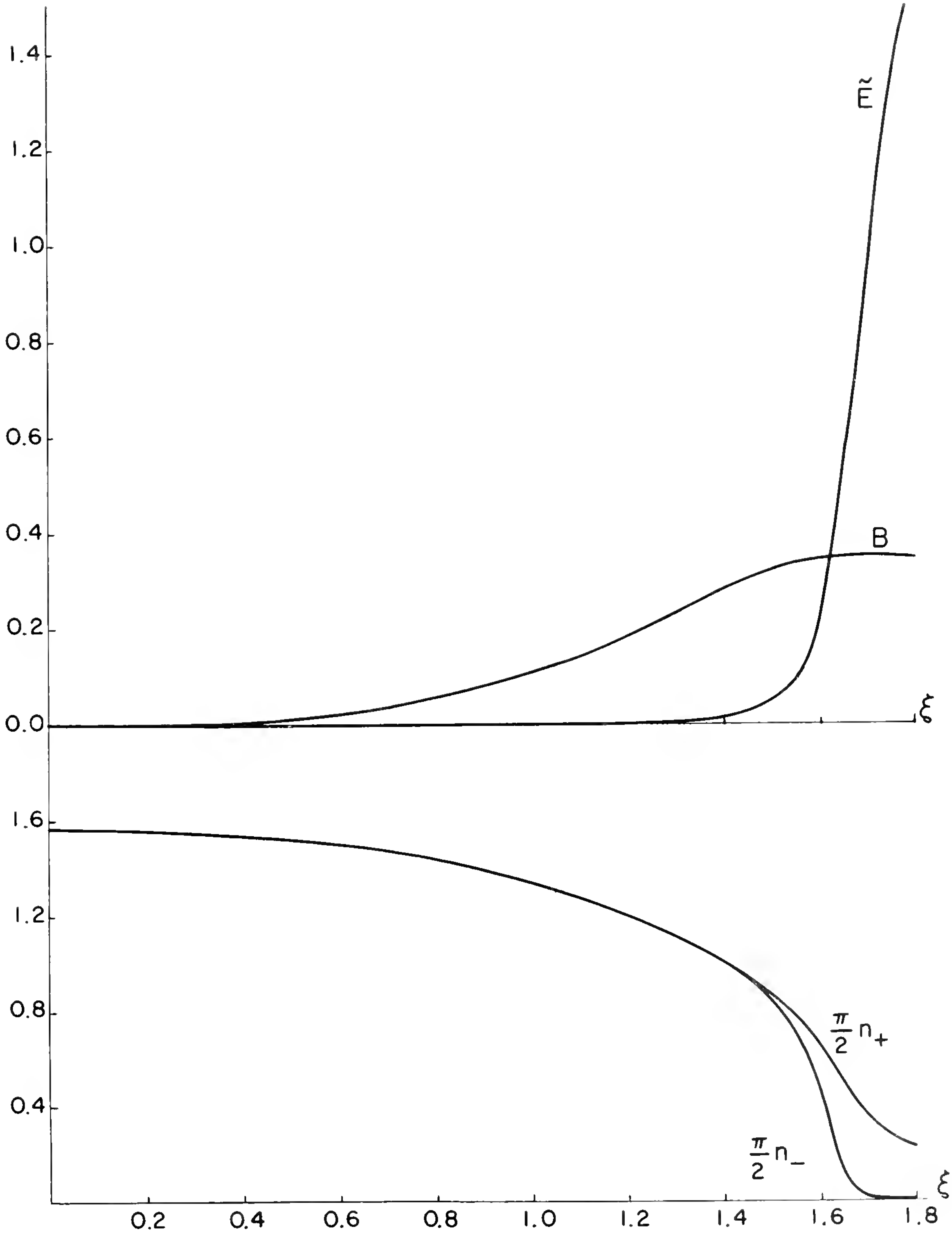

Figure 3b. Solution for $m_-/m_+ = 1/100$; $\bar{v}/c\sqrt{2} = 1/50$.

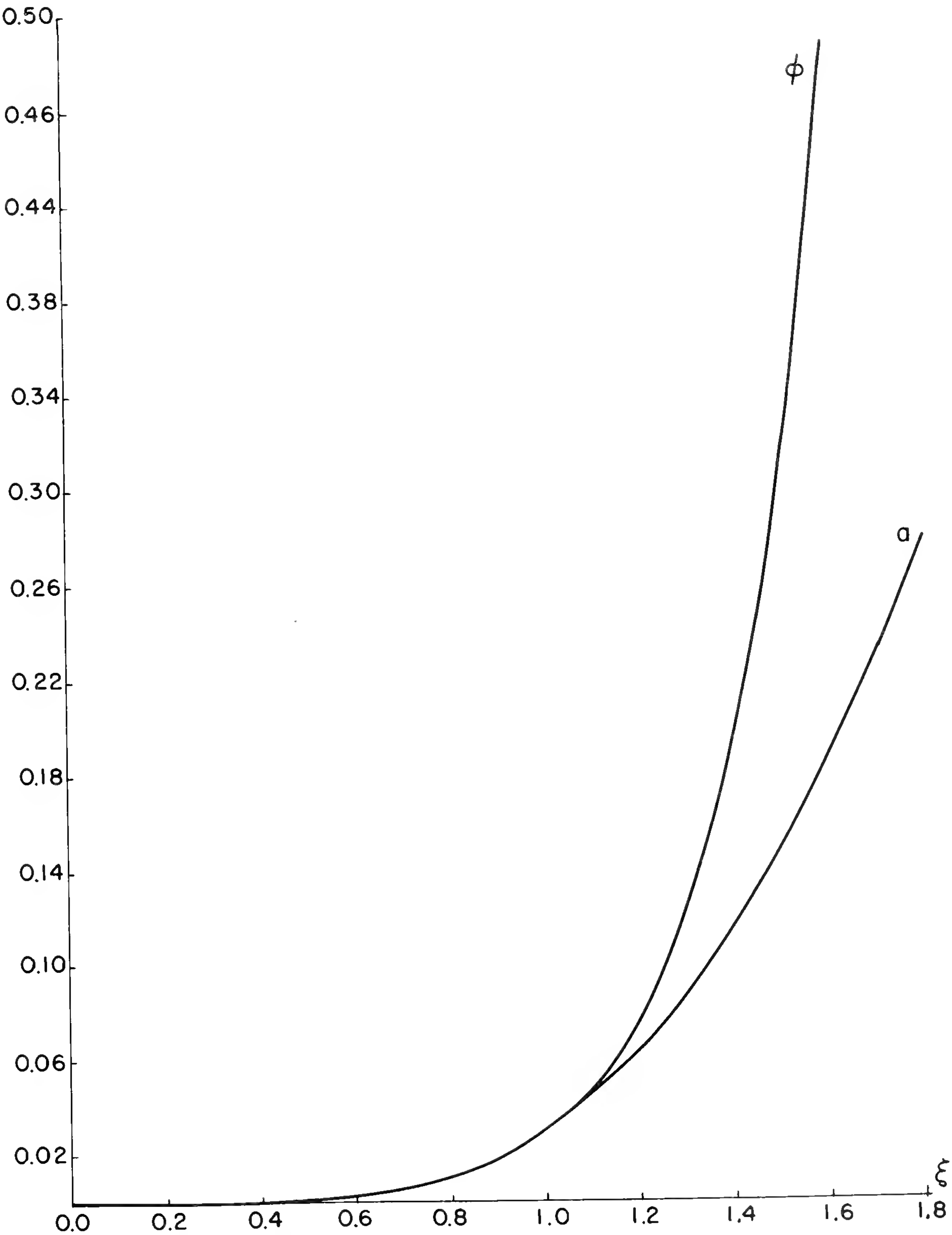

Figure 4a. Solution for $m_-/m_+ = 1/100$; $\bar{v}/c\sqrt{2} = 1/10$.

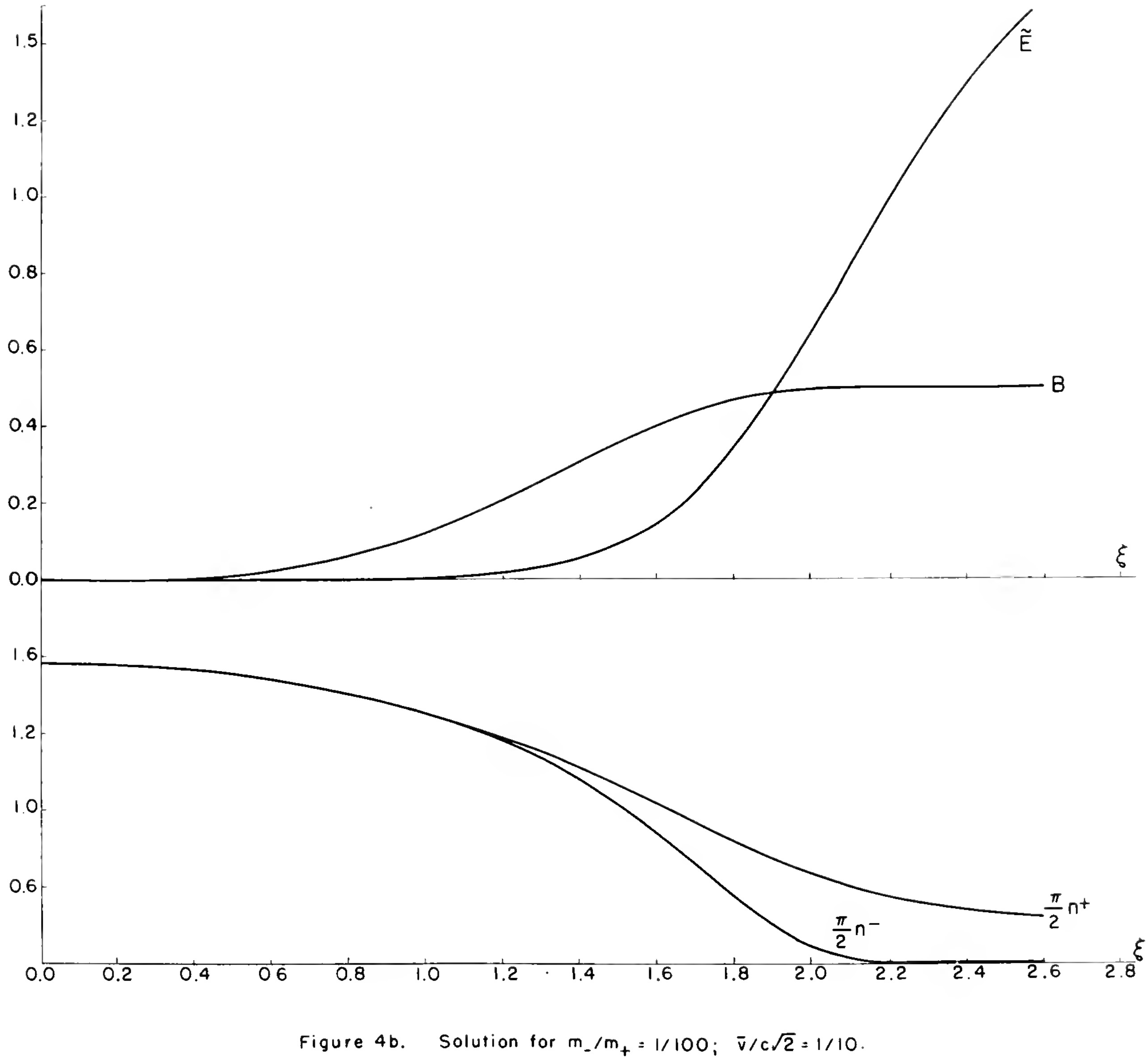

Figure 4b. Solution for $m_-/m_+ = 1/100$; $\bar{v}/c\sqrt{2} = 1/10$.

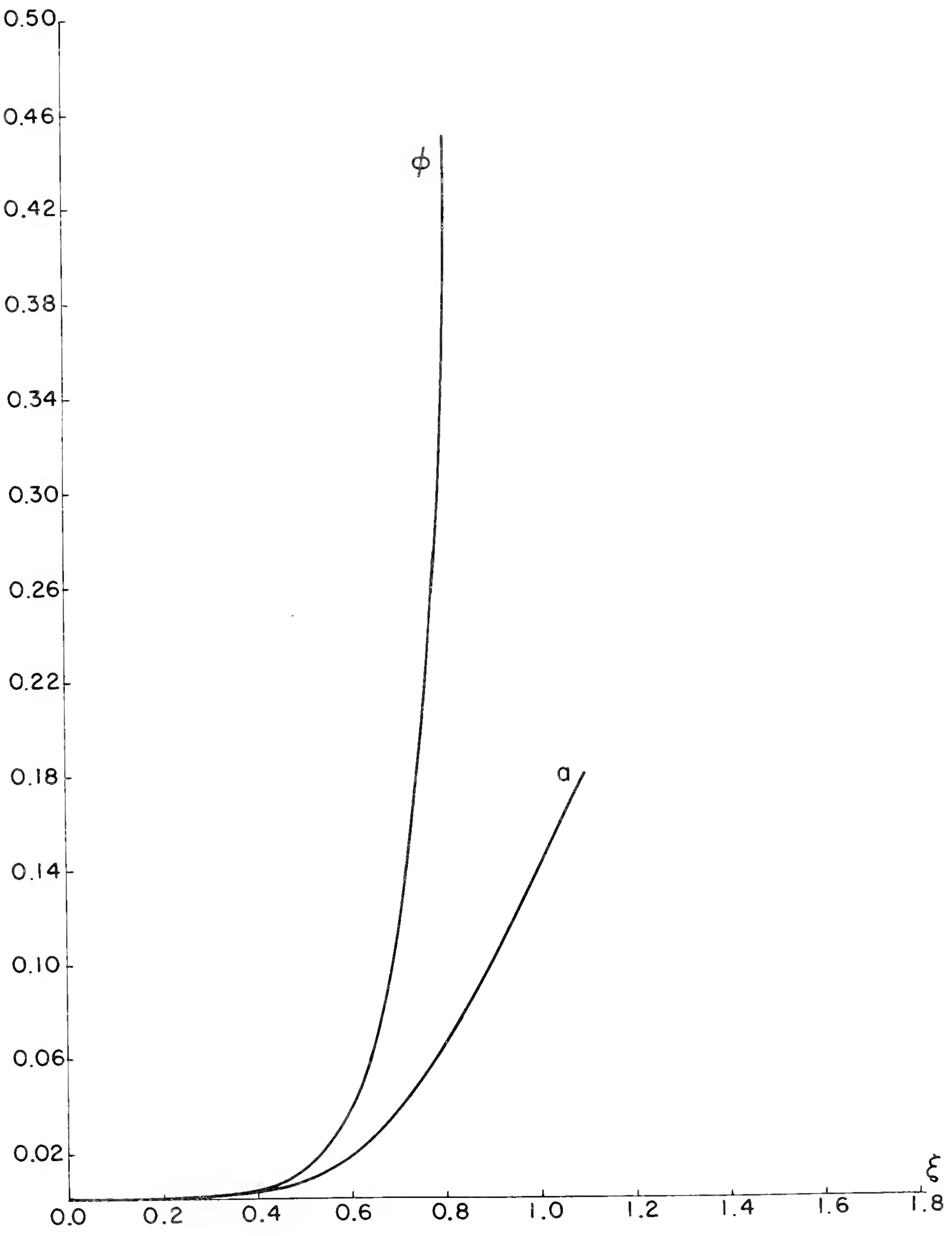

Figure 5a. Solution for $m_-/m_+ = 1/1830$; $\bar{v}/c\sqrt{2} = 1/50$.

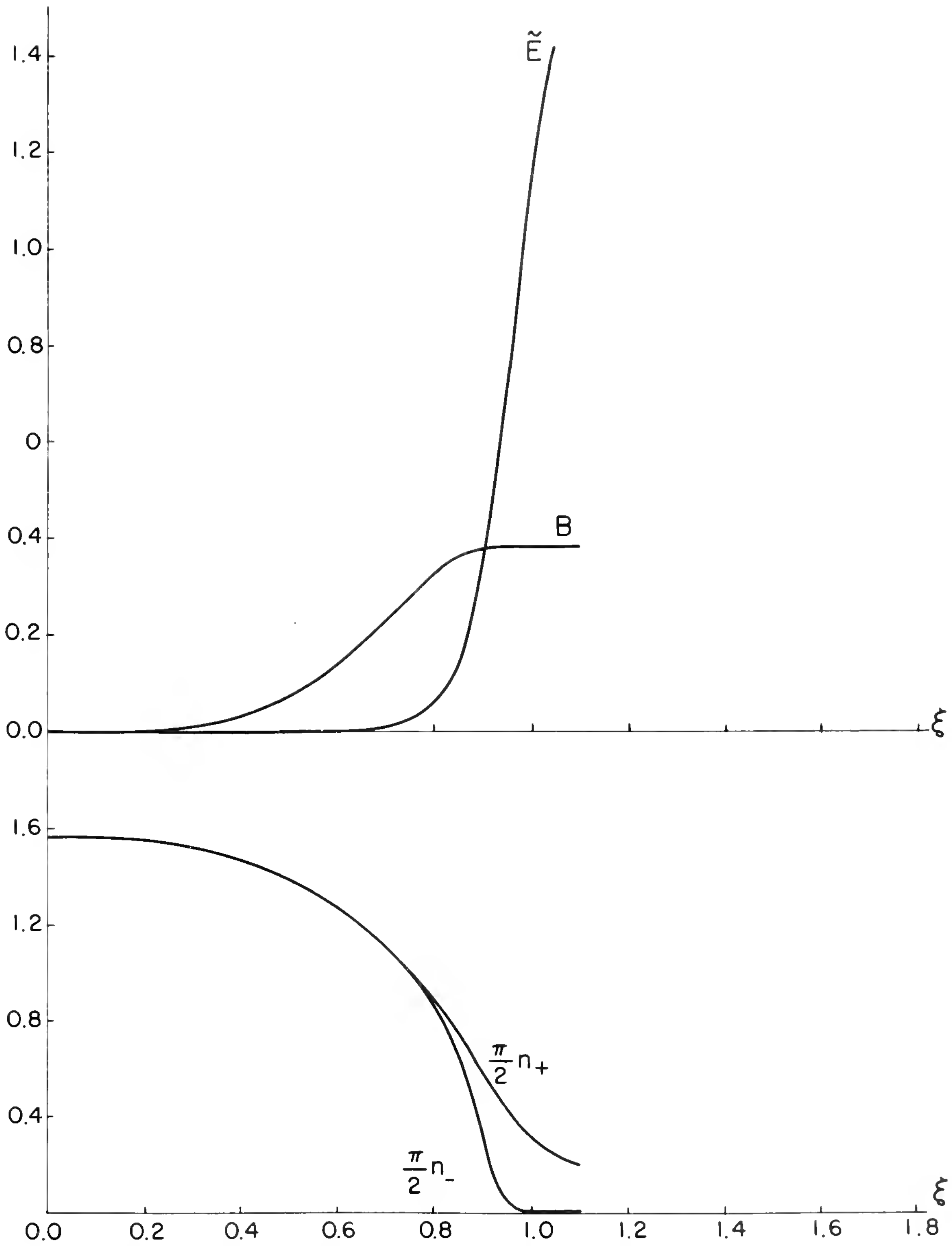

Figure 5b. Solution for $m_-/m_+ = 1/1836$; $\bar{v}/c\sqrt{2} = 1/50$.

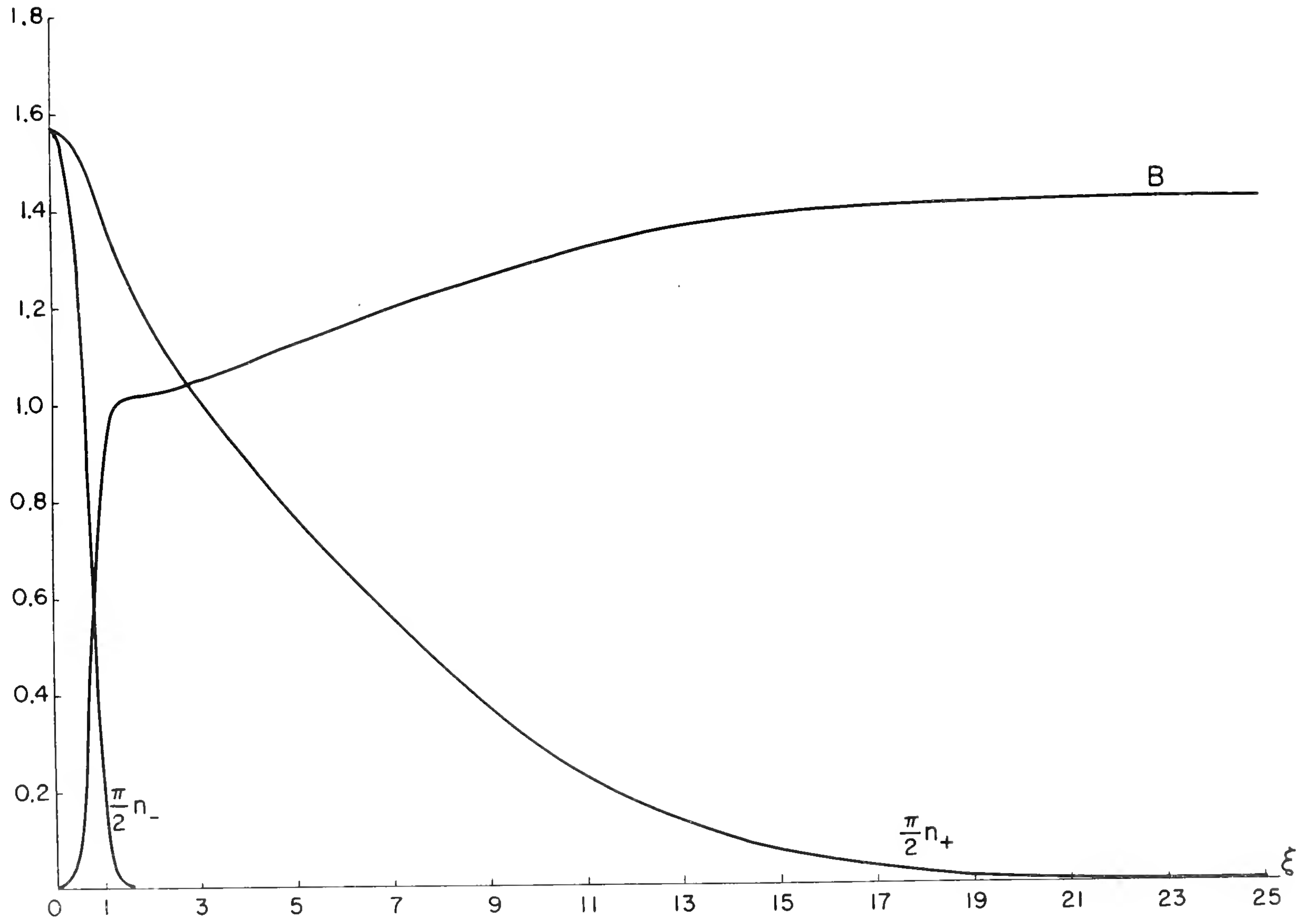

Figure 6. Charge neutral solution with trapped stationary electrons.